

8300

GCSE MATHEMATICS

Foundation and Higher tier – Questions by Topic – Mark Scheme

Ratio, proportion and rates of change

Solving direct proportion word problems

v1.0

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt

M	Method marks are awarded for a correct method which could lead to a correct answer.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied
B	Marks awarded independent of method.
ft	Follow through marks. Marks awarded for correct working following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
M dep	A method mark dependent on a previous method mark being awarded.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
oe	Or equivalent. Accept answers that are equivalent. e.g. accept 0.5 as well as $\frac{1}{2}$
[a, b]	Accept values between a and b inclusive.
3.14...	Allow answers which begin 3.14 e.g. 3.14, 3.142, 3.1416
Use of brackets	It is not necessary to see the bracketed work to award the marks.

Examiners should consistently apply the following principles

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a student has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the student. In cases where there is no doubt that the answer has come from incorrect working then the student should be penalised.

Questions which ask students to show working

Instructions on marking will be given but usually marks are not awarded to students who show no working.

Questions which do not ask students to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Students often copy values from a question incorrectly. If the examiner thinks that the student has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Q1.

Any valid conversion seen, eg

10 (cm) = 4 (inches)

25 (cm) = 10 (inches)

30 (cm) = 12 (inches)

Numbers may be marked next to graph

M1

150 (cm) = 60 (inches)

or

75 (inches) = [185, 190] (cm)

or

75 : 150 = 1 : 2 and inch : cm = 1 : 2.5

or

eg $150 \div 30 = 5$ and $75 \div 12 = 6 \dots$

May use any value [60, 75] (inches) correctly converted to cm to show it is not enough

eg 70 inches = 175 cm

A1

Correct conclusion with appropriate values stated

eg No and 60

or No and [185, 190]

or No and each inch needs 2.5 cm and there are only 2

oe

Strand (iii) Allow Q1ft if M1A0 awarded, an arithmetic error made in calculating conversion of 150 cm or 75 inches and a correct conclusion reached for their values. Must be using correct conversions throughout

Q1ft

Alternative method

Divides 150 and 75 by a common factor of at least 5

eg $150 \div 10 = 15$ and $75 \div 10 = 7.5$

M1

Reads off accurately for one of their values eg 15 cm = 6 inches

or

Draws lines across and down accurately for both values

A1

Correct conclusion comparing their scaled value and graph value or comparing their pairs of lines

Strand (iii) Allow Q1ft if M1A0 awarded, an error made in reading value and correct conclusion reached for their values

M1

Additional Guidance

Note that the list for Q1 are only examples, there are many other possible valid conclusions

eg1 70 inches = 175 cm so 150 cm is not enough

eg2 $150 \div 30 = 5$ and $75 \div 12 = 6 \dots$ so No because need 6 times and only 5.

They must be using a correct conversion for all parts of their answer to qualify for the Q mark. Allow arithmetic errors only.

[3]

Q2.

Correct method to find 1%, 2%, 5%, 10%, 100% or 840% of the number

M1

Fully correct method

M1dep

600

A1

Additional Guidance

Up to M2 may be awarded for multiple attempts if no answer chosen

[3]

Q3.

$18 \div 3$ or 6

or

18×5 or 90

or

$\frac{5}{3}$ oe

M1

30

A1

Additional Guidance

$18 \times 10 \div 6$ with incorrect or no answer

M1A0

Decimals for $\frac{5}{3}$ must be correct to 1dp or better (ie 1.7, 1.67, etc)

$18 \div \frac{3}{5}$ is M1 but $\frac{3}{5}$ alone is M0

[2]

Q4.

Alternative method 1

3×4.5 or 13.5

or 3×4500 or 13500

oe

M1

their $13.5 \div 10 \times 200$

their $\frac{13500 \div 10 \times 200}{1000}$

or

oe

M1dep

270

SC1 digits 27

A1

Alternative method 2

$(200 \times 4.5) \div 10$ or 90 (ml)

oe

M1

their 90×3

oe

M1dep

270

SC1 digits 27

A1

Alternative method 3

$$200 : 10000 \text{ or } \frac{1}{50}$$

$$\text{and } \frac{1}{50} \times 3 \text{ or } 0.06$$

oe

M1

$$\text{Their } 0.06 \times 4.5 \times 1000$$

oe

M1dep

270

SC1 digits 27

A1

Additional Guidance

Students may convert wrongly to millilitres using a factor of 10 (ie 450) then convert back using the same 'wrong' factor to get the correct answer. Allow this, as the method is valid.

However, partial marks cannot be awarded if a wrong conversion factor is used but if digits 27 seen allow SC1

(1 gallon =) 45 millilitres

(3 gallons =) 135 millilitres

135 millilitres $\div 10 = 13.5$ litres

$$13.5 \div 10 \times 200 = 270$$

M1, M1dep, A1

(1 gallon =) 45 millilitres

(3 gallons =) 135 millilitres

$$135 \div 10 \times 200 = 2700$$

SC1

If a 'build up' method is used to get millilitres equivalent to 13.5 litres then it must be fully correct to get the M1dep

13.5

M1

$$10 = 200, 1 = 20, 3 \times 20 = 60, 0.5 = 10$$

M1dep

$$200 + 60 + 10 = 270$$

A0

13.5

M1

$$10 = 200, 1 = 20, 3 = 60, 0.5 = 10$$

M0

$$200 + 20 + 60 + 10 = 290$$

A0

Gallons	1	$\times 3$	3	
Litres	4.5	10	13.5	13.5
Lawn feed (ml)		200	$\times 1.35$	270

M1

M1dep

A1

[3]

Q5.

- (a) $300 \div 4$ or 75
or 300×1.5
2 cakes = $300 \div 2$ or 2 cakes = 150
or
12 cakes = 300×3 or 12 cakes = 900
oe
any correct scaling

M1

450

A1

- (b) (1.5 kg =) 1500 (g)
or 300 g = 0.3 kg or 150 g = 0.15 kg
seen or implied

B1

their 1500 $\div$ their 75
or 6 (+) 6 (+) 6 (+) 2
or 5×4 or 4 (+) 4 (+) 4 (+) 4 (+) 4
oe

M1

20

SC2 14 cakes from 1050g

A1

Alternative method

(1.5 kg =) 1500 (g)
or 300 g = 0.3 kg or 150 g = 0.15 kg
seen or implied

B1

Build up method to total number of cakes from their 1500 with one error

build up values if correct:

4 cakes = 300(g)

8 cakes = 600(g)

12 cakes = 900(g)

16 cakes = 1200(g)

M1

20

SC2 14 cakes from 1050g

A1

Additional Guidance

1500(g)

4 cakes = 300(g)

8 cakes = 600(g)

16 cakes = 900(g) (one error)

24 cakes = 1500(g)

Answer 24 cakes

is B1M1A0

1000(g) uses incorrect total of flour (misread)

4 cakes = 300(g)

8 cakes = 600(g)

12 cakes = 900(g)

Answer 12 cakes (one error – should be 13 cakes)

is B0M1A0

[5]

Q6.

(a) 600

B1

(b) $900 - 860$ or $860 + 40 = 900$ or 40

or

$0.9 - 0.86$ or $0.86 + 0.04 = 0.9$ or 0.04

Condone 860 – 900

oe

Condone incorrect or missing units

M1

40 grams or 0.04 kg

SC1 940 g or 0.94 kg

A1

Additional Guidance

If you see $860 + 40 = 900$ but then further work to build up to eg 1800, mark the whole method and the only mark available is the SC1.

Once 40 g or 0.04 kg seen, ignore any attempt to change units.

40 g seen in working but then 40 on ans line – condone. M1A1

[3]

Q7.

$$x + x + 30 = 110$$

(x representing Ann)

M1

$(x =)$ 40

A1

$$\begin{array}{r} \text{their } 40 + 30 \\ \hline 110 \end{array}$$

oe fraction $\frac{7}{11}$

A1ft

Alternative method 1

$$x + x - 30 = 110$$

(x representing Tom)

M1

$(x =)$ 70

A1

$$\begin{array}{r} \text{their } 70 \\ \hline 110 \end{array}$$

oe fraction $\frac{7}{11}$

A1ft

Alternative method 2

$$(110 - 30) \div 2 \text{ or } 80 \div 2$$

M1

40

A1

$$\begin{array}{r} \text{their } 40 + 30 \\ \hline 110 \end{array}$$

oe fraction $\frac{7}{11}$

A1ft

Alternative method 3

$(110 + 30) \div 2$ or $140 \div 2$

M1

70

A1

$$\frac{\text{their } 70}{110}$$

$$\text{oe fraction } \frac{7}{11}$$

A1ft**Alternative method 4**

$110 \div 2 + 15$ or $55 + 15$

M1

70

A1

$$\frac{\text{their } 70}{110}$$

$$\text{oe fraction } \frac{7}{11}$$

A1ft**Additional Guidance**

$$\frac{85}{110} \quad \text{is M0 A0 A0ft}$$

[3]**Q8.**

$10.74 \div 6 \times 11$

or

1.79 seen

$$\text{oe eg } 2 \times 10.74 - 10.74 \div 6$$

M1

19.69

A1**Additional Guidance**

$6 \div 10.74 = 1.79$ (recovered)

M1

$6 \div 10.74$

M0**[2]****Q9.*****This question has been adapted to better suit the style of the current specification***

(a) $5.9(0) \div 2$ or 2.95

M1

their 2.95×5

or

their $2.95 \times 3 + 5.9(0)$

M1dep

14.75

A1

(b) 500×8000 or $4\,000\,000$

or

$500 \div 1000$ or 0.5

or

500×8

or
 $8000 \div 2$ or 8000×0.5
 or 1 litre = 1000 millilitres seen or implied

M1

4000

A1

[5]

Q10.

850×1.18 or 1003

oe

$(990 + 15) \div 1.18$

or $990 \div 1.18$ or 838.9(...)

M1

1003 and 1005

or 2

851(...) or 852

or 1(...)

A1

Laura and 1003 and 1005

or Laura and 2

or UK and 1003 and 1005

or UK and 2

or Laura and 851(...) or 852

or Laura and 1(...)

or UK and 851(...) or 852

or UK and 1(...)

*Strand (iii) decision to match **their** calculation*

*ft **their** comparison of values with M1 scored, both values must be in the same currency*

Q1ft

Additional Guidance

Accept name, country or price (e.g. the (£)850 saddle) for final answer

$990 \div 1.18 = 838.(\dots)$, Steve (or Holland)

M1A0Q1ft

$990 \div 1.18 = 838.(\dots)$, $15 \div 1.18 = 12.(\dots)$, $838 + 12 = 850$, they both cost the same

M1A0Q1ft

Laura with no valid working

For the Q mark, follow through *their* comparison of values with M1 scored, but both values must be in the same currency and one of the values used in the comparison must be from the M1 that was awarded.

M0A0Q0

[3]

Q11.

Alternative method 1

$3.2(0) \div 5$ or 0.64

or

0.29×3 or 0.87

oe eg working in pence

M1

$3.2(0) \div 5 \times 12 + 0.29 \times 3$

or

$$7.68 + 0.87$$

*oe eg working in pence
must be consistent units*

M1dep

$$8.55$$

condone £8.55p

A1

Alternative method 2

$$12 \div 5 \text{ or } 2.4$$

or

$$5 \div 12 \text{ or } 0.41(6\dots) \text{ or } 0.417$$

$$\text{or } 0.42$$

M1

$$3.2(0) \times \text{their } 2.4 + 0.29 \times 3$$

or

$$3.2(0) \div \text{their } 0.41(6\dots) + 0.29 \times 3$$

*oe eg working in pence
must be consistent units*

M1dep

$$8.55$$

condone £8.55p

A1

Additional Guidance

Inconsistent units may be recovered in final answer
7.68 in working implies M1

[3]

Q12.

$$\frac{18}{25} (\times 100) (= 72\%) \text{ or } \frac{72}{100}$$

$$\text{or } 18 \div 25 \text{ or } 0.72 \text{ oe}$$

Working with marks lost

$$\frac{7}{25} (\times 100) (= 28\%) \text{ or } \frac{28}{100}$$

$$\text{or } 7 \div 25 \text{ or } 0.28 \text{ oe}$$

M1

$$\frac{30}{40} (\times 100) (= 75\%) \text{ or } \frac{75}{100}$$

$$\text{or } 30 \div 40 \text{ or } 0.75 \text{ oe}$$

$$\frac{10}{40} (\times 100) (= 25\%) \text{ or } \frac{25}{100}$$

$$\text{or } 10 \div 40 \text{ or } 0.55 \text{ oe}$$

Note: 18×8 and 30×5 implies M2

M1

Test B and correct pair compared

(30 out of 40)

e.g.

0.72 and 0.75

72 and 75

144 and 150 (marks out of 200)

28 and 25 (% incorrect)

Alternative Method

$$18 \div 25 \text{ or } 30 \div 40$$

$$18 \div 25 \times 40 \text{ or } 30 \div 40 \times 25$$

Test B and correct pair compared
(30 out of 40)

e.g.
28.8 (and 30)
or 18.75 (and 18)

Q13.

(a) $1.5 + 7.5 (= 9)$

9 seen as denominator

$$\frac{1.5}{\text{their } 9} \text{ or } \frac{3}{18}$$

oe

$$\frac{1}{6}$$

0.16... or 0.17 implies M1M1A0

$$\text{SC2 } \frac{5}{6}$$

$$\text{SC1 } \frac{1}{5} \text{ or } \frac{4}{5}$$

(b) 12 litres = 75% oe

$$3(2 + x) = 12 \text{ or } 6 + 3x = 12$$

or $12 \div 3$

$$\frac{x+2}{x+2+12} = \frac{1}{4} \text{ or } 4(x+2) = x+2+12$$

$$\text{or } 4x + 8 = x + 2 + 12$$

$$\frac{B}{B+12} = \frac{1}{4} \text{ or } 4B = B + 12$$

4 litres = 25%

or 4 litres = $\frac{1}{4}$

or 16 litres = 100%

or $\frac{4}{16}$

oe

$$2 + x = 4 \text{ or } 3x = 12 - 6$$

$$4x - x = 2 + 12 - 8$$

$$4B - B = 12$$

(Add) 2 (litres)

A1**M1****M1****A1****[3]****M1****M1dep****A1****M1****M1dep****A1****[6]**

Q14.

(a) $1.6 \times 6 \frac{1}{2}$

10.4 oe

M1

A1

(b) Use or sight of 4.5(4) litres = 1 gallon oe

B1

A correct single step calculation

50×1.6 or 80

$100 \div 1.6$

$100 \div 5.5$

or $5.5 \div 100$

$5.5 \div 4.5$

or $4.5 \div 5.5$

$50 \div 4.5$

or $4.5 \div 50$

M1

A **different** correct single step calculation

$50 \times 1.6 \div 4.5$

$80 \div 4.5$

$4.5 \div (50 \times 1.6)$

$4.5 \div 80$

or

$(50 \div 4.5) \times 5.5$

$100 \div (5.5 \div 4.5)$

A correct two-step calculation

$(100 \div 1.6) \div 5.5$

$100 \div 5.5 \times 4.5$

$(100 \div 1.6) \times 4.5$

M1dep

Two matching values

(May be rounded)

(May be multiples of figures listed)

Units	Manu facturer	My Car
Km per litre	18.18	17.7*
Km per gallon	81.8*	80
Litres per 100 km	5.5 (given)	5.625*
Litres per km	0.055	0.05625*
Miles per litre	11.36*	11.1
Litres per mile	0.088*	0.09
Miles per 5.5 litres	62.5	61.1*
Km per 5.5 litres	100 (given)	97.8*
Gallons per mile	0.019..*	0.02
Miles per gallon	51.1*	50 (given)
Gallons per km	0.0122..	0.0125*
Litres per 800 km	44	45*

**these values imply a correct two-step calculation for M2*

Values may be rounded or truncated such that correct comparisons can still be made.

More fuel

A1

Q1

[7]

Q15.

$$\frac{20}{8}$$

or 2.5 seen or implied

$$\frac{8}{20}$$

or 0.4 seen or implied

or 75 + 75 + 37.5 or 187.5

or 50 + 50 + 25 or 125

or 40 + 40 + 20 or 100

or 2 + 2 + 1 or 5

oe

M1

Two from

187.5 or 125 or 100 or 5

For 187.5 allow [187, 188] or 190

A1

187.5 and 125 and 100 and 5

For 187.5 allow [187, 188] or 190

SC1 for [112, 113] and 75 and 60 and 3

A1

[3]

Q16.

$$\frac{20}{40}$$

× 60 (= 30) **or**

$$\frac{20}{40}$$

× 120 (= 60) **or**

$$\frac{20}{40}$$

× 180 (= 90)

oe eg 1 60 ÷ 2

eg 2 60 ÷ 40 (= 1.5) **and** their 1.5 × 20

M1

$$\frac{15}{20}$$

× 60 (= 45) **or**

$$\frac{15}{20}$$

× 120 (= 90) **or**

$$\frac{15}{20}$$

× 180 (= 135)

oe eg 1 180 ÷ 4 × 3

eg 2 60 ÷ 20 (= 3) **and** their 3 × 15

M1

their 30 + their 45

or

their 60 + their 90

or

their 90 + their 135

dep on at least one M1

M1dep

(Sugar) 75

(Butter) 150

(Flour) 225

All 3 correct

SC2 No working with two correct answers

SC1 No working with one correct answer

A1

Alternative

$\frac{20}{40}$ and $\frac{15}{20}$

oe eg 0.5 and 0.75

M1

their $\frac{20}{40} +$ their $\frac{15}{20} (= \frac{5}{4})$

oe eg 1.25

M1

their $\frac{5}{4} \times 60 (= 75)$ **or**

their $\frac{5}{4} \times 120 (= 150)$ **or**

their $\frac{5}{4} \times 180 (= 225)$

oe eg 1.25×60

M1dep

(Sugar) 75

(Butter) 150

(Flour) 225

All 3 correct

SC2 No working with two correct answers

SC1 No working with one correct answer

A1

[4]

Q17.

Alternative method 1

$150 \div 6$ or 25 (1 person)

150 $\times$ 2 or 300 (12 people) or

$\frac{150}{2}$ or 75 (3 people)

M1

their 25×15

their 300 + their 75 or

their 75×5

M1dep

375

A1

Alternative method 2

$15 \div 6$ or 2.5

M1

their 2.5×150

M1dep

375

A1

[3]

Q18.

Alternative method 1

$8400 \div 350$ or 24

oe method to work out 24

M1

their 24×115

M1dep

2760

A1

Alternative method 2

$8400 \div 3 \div 350$ or 8

oe method to work out 8

M1

their $8 \times 3 \times 115$

M1dep

2760

A1

Alternative method 3

$350 \times 3 = 1050$

and $8400 \div$ their 1050

or 8

M1

their $8 \times 3 \times 115$

M1dep

2760

A1

[3]

Q19.

$99.2(0) \div 5 \times 24$

or

19.84×24

or

$4.8 \times 99.2(0)$

or

$2380.8(0) \div 5$

or

11904

25

oe

M1 99.2(0) $\div$ 5 or 19.84 oe

or

24 $\div$ 5 or 4.8 oe

or

24 $\times$ 99.2(0) or 2380.8(0) oe

M2

476.16

A1

Additional Guidance

(One year's broadband cost) 238.08 oe implies M1

[3]

Q20.

$2.40 \div 12$ or $0.2(0)$

or

$240 \div 12$ or 20

or

$30 \div 12$ or 2.5

*oe eg build up in 20s to 240***M1**

$30 \times$ their 0.2 or $30 \times$ their 20

or

$2.4(0) \times$ their 2.5

*oe***M1**

$6(.00)$

A1**Additional Guidance**

Condone 600p

M1M1A1

600

M1M1A0**[3]****Q21.****Alternative method 1**

$270 \div 3$ or 90

or

270×50 or 13 500

or

$270 \times 0.5(0)$ or 135

*oe***M1**

$270 \div 3 \times 50$

or

$270 \div 3 \times 0.5(0)$

*oe***M1dep**

45

A1**Alternative method 2**

Correctly finds units and cost for at least 30 miles

*eg**30 miles = 10 units and 10 units cost (£)5***M1**

Correct method for cost of 270 miles

M1dep

45

A1

Additional Guidance

45.00(p)

M1M1A1

45.0

M1M1A0Accept 0.33(...) or 33.(...) % for $\frac{1}{3}$ Further work eg $45 \div 2$ **M1M1A0**

Embedded answer of 45 but not selected

M1M1A0**[3]****Q22.** $\frac{30}{20}$

or 1.5 seen or implied

or $180 + 90$ or 270or $150 + 75$ or 225or $200 + 100$ or 300or $4 + 2$ or 6

oe

M1

Two from

270 or 225 or 300 or 6

A1

270 and 225 and 300 and 6

A1**[3]****Q23.****Alternative method 1** $260 \times 0.4(0)$ or $104(.00)$

or

 260×40 or 10 400

oe

*cost of claim***M1** $260 \div 52$ or 5

oe

*number of gallons***M1**their 5×5.36 or $26.8(0)$

oe

*dep on 2nd M1**cost of petrol***M1dep**

77.20

A1

Alternative method 2

260 ÷ 52 or 5

oe
number of gallons

M1

52 × 0.4(0) or 20.80

or

52 × 40 or 2080

oe
claim per gallon

M1

their 20.80 – 5.36 or 15.44

or their 2080 – 536 or 1544

dep on 2nd M1
claim per gallon – cost per gallon

M1dep

77.20

A1

Alternative method 3

5.36 ÷ 52 or 0.10...

or 536 ÷ 52 or 10.(...)

cost of petrol per mile

M1

0.4 – their 0.10... or [0.2969, 0.3]

or

40 minus; their 10.(...) or [29.69, 30]

claim per mile – cost per mile

M1dep

their [0.2969, 0.3] × 260

or

their [29.69, 30] ÷ 100 × 260

M1dep

77.20

A1

Additional Guidance

Accept working in pounds or pence for all three method marks

Condone £77.20p

M1M1M1A1

77.2

M1M1M1

Answer £77.2

M1M1M1A0

[4]